Principles of IUT — RIMS '21

§ - IUT - timeline.

[Alien] | [EssLgc]

2010 - RIMS '15 Oxford | '20 Pub.

2012 - Online '16 RIMS | '21 ExpHor

1/ Discovery

[IUT1-4]

2/ Expansion

[P] pub [Hoshi] TBA

[ExpEst] abc + FLT

IUT '21 = New geometrization.

"Diophantine Anab. Geom. of Monoids

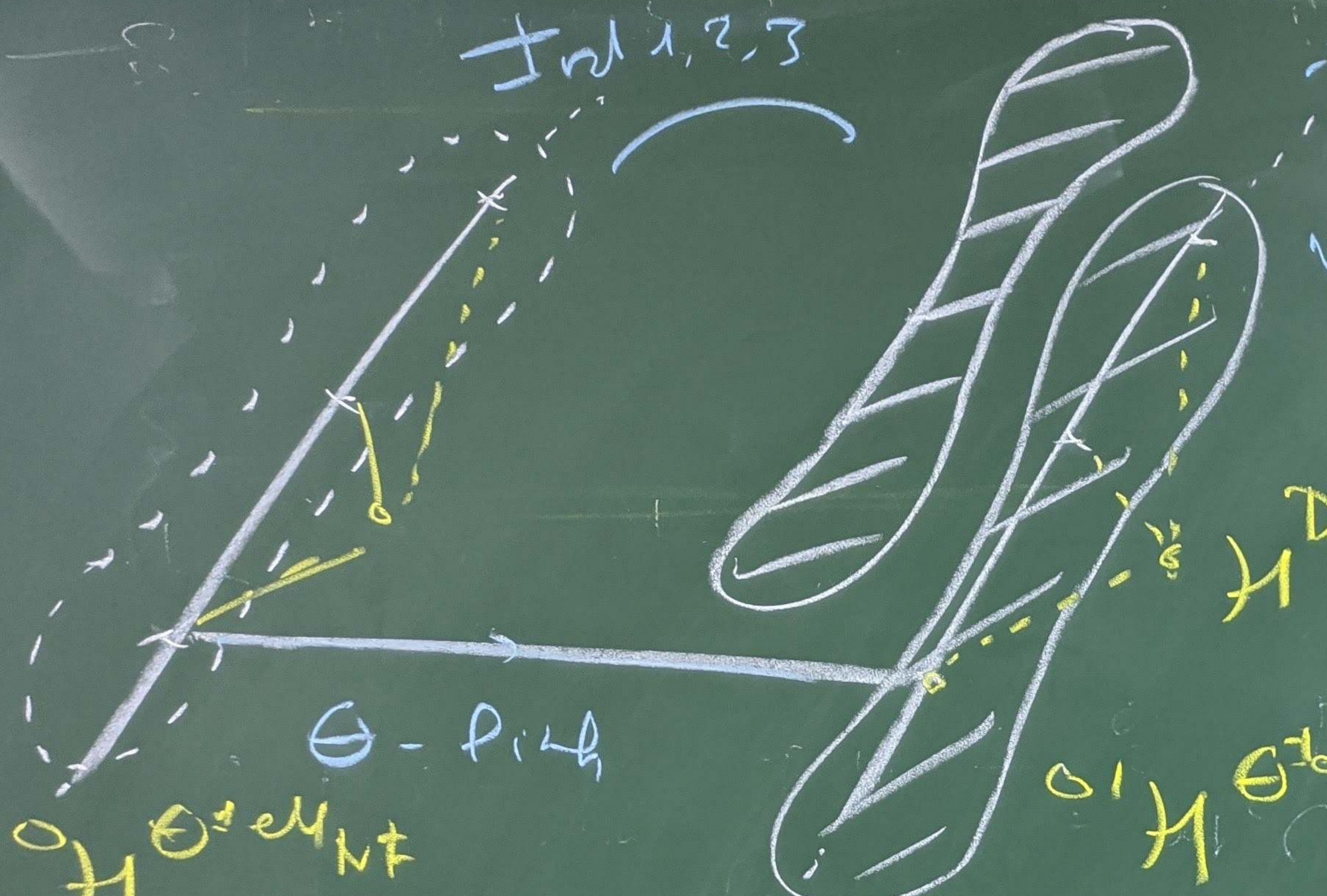
· R-pt, R_v-pt Gal. $\pi_1^{(tp)}$, Kummer Ring $(R,+,\times) \rightsquigarrow$ Mon

· heights Abs $\pi_1 \not\to$ Gal. Spec(A) $\mathcal{O}^{\triangleright}$ Value

$\mathcal{O}^{\times}$ unit

$\times \Downarrow < \mathbb{R}_+$ α: Tell!, c, 3. Drift

§ IUT-space: log-Θ-lattice of HT-theatres



H.R.A $\equiv$ ~ $\underline{\underline{U}}^{\log}$

$\mathbb{R}_{< -\log|\Theta|}$

$\mathbb{R} \ni -\log|q|$

$^{\circ\circ}\mathcal{H}^{\Theta^{\pm ell}NF}$

$^{\circ|}\mathcal{H}^{\Theta^{\pm ell}NF}$ of P. Strips

$^{\dagger}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}$ — p. sh

Θ

$^{\dagger}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}$ — prim strip

$^{\dagger}\xi^{\Vdash}_{mod}$ ~ "An.t! L.B"

$\{q^{j^2}\}_j^N$ $\longmapsto$ $\{G_v \cap \mathcal{O}^{\times\mu} \cdot \{q^{j^2}\}_j^N\}_V$

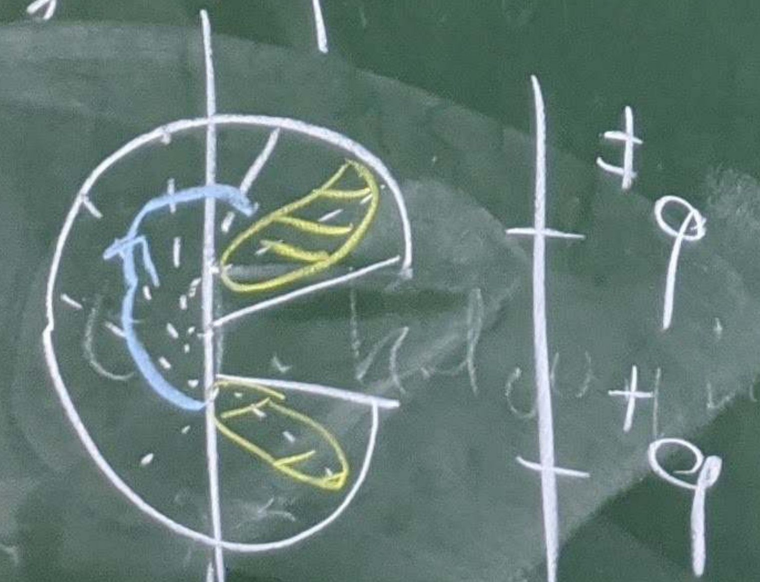
$\parallel$

$^{\dagger}\{p_v\}_V$ ~ loc. glb.

Θ-pilot

q-pilot

② This a path in a R.S? $\neq$ Iterative process!



§ - Geom. of $\Theta^{\pm ell}$NF HT

Gluing - ΘNF & Θ^{ell} / bases, capsul

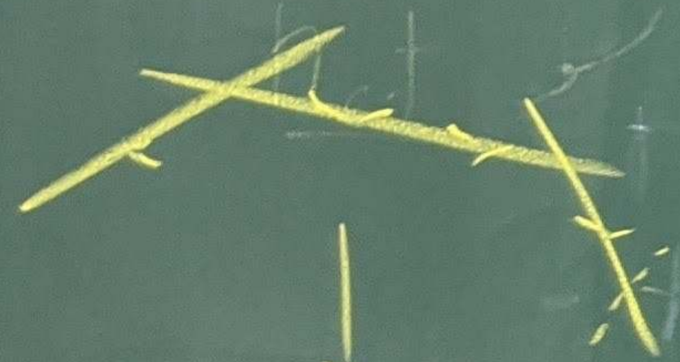
a/ D-NF ($\mathcal{D}_{>} \leftarrow \mathcal{D}_{J} - \mathcal{D}^{\Theta}$)

$J = \mathbb{F}_l^{*}$ loc glob

$v \in V^{bad}$ $\{\mathcal{D}_v\}$ $\{\pi_1^{tp}(\underline{X}_v)\}_V$ $\pi_1^{et}(\underline{C}_K)$

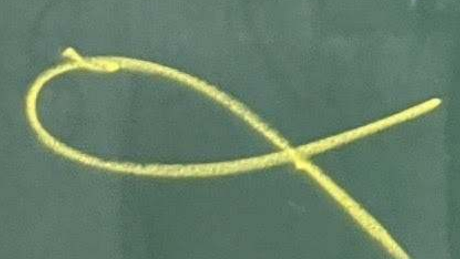
K (Arith, $\boxtimes$) $Aut(\underline{C}_K) \subset Gal(K/F_{mod}) \cong \mathbb{F}_l^{*}$

② Sync. Intra Aut. of $\{G_v\}_V \subseteq Gal(K/F_{mod})$.



b/ D-$\Theta^{\pm ell}$ ($\mathcal{D}_{>} \leftarrow \mathcal{D}_{T} - \mathcal{D}^{\Theta^{\pm}}$)

$T = \mathbb{F}_l^{\rtimes \pm}$



$\mathcal{D}_v$ $\{\pi_1(X_v)\}$ $\pi_1^{et}(\underline{X}_K)$

(Geom, $\boxplus$) $Aut(\underline{X}_K) \cong \mathbb{F}_l^{\rtimes \pm}$

Nb: , $X \in \mathcal{M}_{1,1}(K)$ w/ $\mathcal{O}_v$ ~ temp. Anab. Geo.

, π_1^{tp} vs π_1^{et} Anab. Geo ~ OK

, Goal is GMS + GCG,

$\hookrightarrow$ $\Theta^{\pm ell}_{NF} = \Theta^{\pm ell} + \Theta^{NF}$ [Glutching]

$\Theta^{\pm ell}$: $\rightsquigarrow$ Data a/ Copsul

$\rightsquigarrow$ Generate (Frob) $D = \{\Pi_v\}_{\underline{V}}$, $\mathcal{F}_v = \{\Pi_v \curvearrowleft \mathcal{O}_v^{\triangleright}\}_{\underline{V}}$

$D^{\vdash} = \{ G_v \}_{\underline{V}}$

Θ^{NF} : $\rightsquigarrow$ Data b/ Copsul,

$\sim \mathcal{F}^{\Vdash}_{mod} = (\mathcal{C}^{\Vdash}_{mod}, \mathrm{Prime}(\mathcal{C}^{\Vdash}_{mod}) \cong \underline{V}, \{\rho_v\}_{\underline{V}})$

Dioph height degree.

Aut C.B on $[\mathrm{Sp}\,\mathcal{O}_K / \mathrm{Gal}(K/F_{mod})]$

loc-glob

$\rightsquigarrow$ Generate $\mathcal{F}^{\Vdash \blacktriangleright \times \mu}$ $\ni G_v \curvearrowright \mathcal{O}_v^{\times\mu} \times \{q_v^{j^2}\}^{\mathbb{N}}$

§ - Int. Sym. ("Int" Mono-An.)

, **Ind 1** Mono. Anab. Transport (Étale)   [AbTop III]

$\exists$ Fun. Alg. $\quad G_v \sim \mathcal{O}_v^\times,\ \Lambda(\mathbb{A})$ ⇝ Cyclotom.

$G_v \curvearrowright \Pi_v \sim \hat{\mathbb{Z}}^\times$-orbit

$M \simeq \mathcal{O}^\triangleright_{\mathbb{A}}(G_v) \qquad M \simeq \mathcal{O}^\triangleright_{\mathbb{A}}$

, Transport (Alien 2.12.2) $\curvearrowleft \mathrm{Aut}(G_k)$

EL. $\quad H^1(G_k, \Lambda(G_k)) \simeq H^1(G_k, \Lambda(G_k))$

$$\begin{array}{ccc} H^1(G_k, \Lambda(G_k)) & \simeq & H^1(G_k, \Lambda(G_k)) \\ \uparrow u & \curvearrowleft \hat{\mathbb{Z}}^\times & \downarrow u^{-1} \\ {}^\dagger\mathcal{O}^\times & \text{- - - - - - -} & {}^\ddagger\mathcal{O}^\times \end{array}$$

Frob.

⇝ **Ind 1** = $\mathrm{Aut}(G_k)$-Indeterm.

Indet. $\mathrm{Aut}(G_k \curvearrowright \mathcal{O}_k^\times) = \mathrm{Aut}(G_k) \times \hat{\mathbb{Z}}^\times$ prof. top.

Since $\hat{\mathbb{Z}} \curvearrowright \mathrm{End}(\mathcal{O}_k^\times = \varprojlim_n \mathcal{O}_n)$

Ind2: $\times\mu$-Kummer str (Fab w EL)

In $\mathcal{F}^{\vdash\times\mu}$: $G_v \cap \mathcal{O}^{\times\mu}$ has a

$\times\mu$-str: $\{\mathcal{I}^{u}_{H}\}_{H \overset{op}{\leq} G_v} := \{\mathrm{Im}[\mathcal{O}_v^{\times H} \cdots \mathcal{O}^{\times\mu}]\}_{H \leq G_v}$

At the prime strip level: $G_v \cap \mathcal{O}_v^{\times\mu}$

Fab

$\mathbb{Z}_p^\times$

$\Pi_v \cap \mathcal{O}^{\triangleright}$ $\quad \mathcal{F} \quad \sim \quad \mathcal{F}^{\vdash\times\mu}$

$\parallel$

Not $\cap$ $\quad \{ \quad (\times) \quad \{ \ \cap$ Ind2: $\mathrm{Aut}^{\times\mu}_{G_v}(\mathcal{O}_v^{\times\mu}) = \mathrm{Im}(\hat{\mathbb{Z}}^\times)$

(Tsujimura)

Π_v $\quad \mathcal{D} \quad \sim \quad \mathcal{D}^{\vdash}$

EL.

G_v

Holomorphic

Mono-Anal.

NB: Ind (×) Apply z.g. shell $\mathcal{I}_\square$

- Mono-G: M^{G} = rigid "cxl/disc/cst" [EtTh]
- Tsujimura: for $k = \mathbb{Q}_p$, $\mathrm{Aut}(G_k) = \{1\}$ $\Longleftarrow$ [Hoshi]

§ Multiradiality - Ind3 - Sym ("Ext")

$^{\dagger}(x) \overset{Z}{\sim} {}^{\ddagger}(x)$ $\quad \cup x$ log-shell

$$\mathcal{I}_v = (2p_v)^{-1} \log_v(\mathcal{O}_{\bar{k}}^{\times\mu})$$

Rep $\boxplus$, $\boxtimes$ | Ring

$$\begin{array}{ccc} \equiv \wr & & \wr \equiv \\ {}^{\dagger}\mathcal{HT} & \xrightarrow{\Theta} & {}^{\ddagger}\mathcal{HT} \\ \downarrow & & \downarrow \\ {}^{\dagger}\mathfrak{E} & \xrightarrow{\sim} & {}^{\ddagger}\mathfrak{E} \end{array}$$

$\mathfrak{E} = (G_v \curvearrowright \mathcal{O}^{\times\mu}, \mathcal{I}^{Q})$

$\cdots\ {}^{n}\mathcal{O}^{\triangleright} \cdots\cdots {}^{n+1}\mathcal{O}^{\triangleright} \cdots$. co-chain of $\{\log_n\}$

$\quad \boxplus\ \boxplus$ | $\boxplus$ Hy

$\mathcal{O}^{\times\mu}_{\boxtimes} \xrightarrow{\log^n} k$

. $\mathcal{I}$ ut-container

$\mathbb{I} \in \{\mathrm{Et}, \mathrm{Frb}\} \times \{\mathrm{Mon}, \mathrm{Hol}\}$

$\mathcal{I}_{\mathbb{I}} \leq k$ [Ab Tuj $\mathbb{I}$]'

$\mathcal{E}_{\boxplus}$ - Mon

$(k, \times)$ of

$k^{\times}$ Auto.

$\mathcal{O}^{\triangleright}_{\boxtimes} \xrightarrow{\log} \mathcal{O}^{\triangleright}_{\boxplus}$

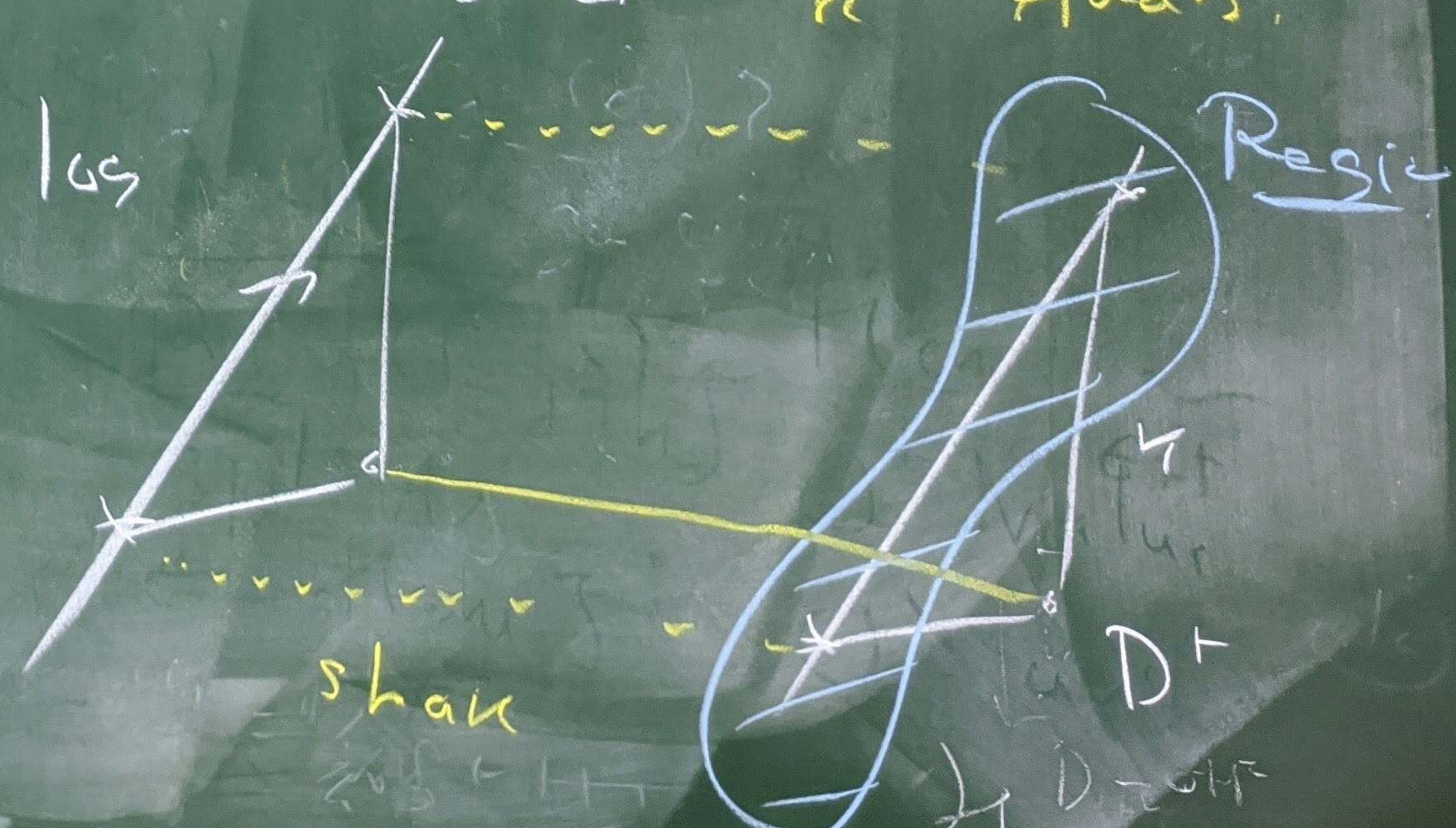
, $\mathcal{I}_{\mathbb{I}}^{\mathbb{D}}$ $\otimes$-proc-packet

Labcusp const unit Π_v

$\mathcal{O}^{\triangleright} — \mathcal{O}^{\triangleright}(\Pi_v^{\mathbb{H}})$ unit log-cusp.

~ Ind3: Log-Kummer up. conn. (Hol & Frb)

$$\underbrace{\mathcal{I}^{\mathbb{D}}(\mathbb{D}^{\vdash})}_{\log\ \text{Mon-Et}} \geq \underbrace{\bigcup k({}^{n}\mathcal{F}^{G})}_{\text{Kum}\ \text{Hol. Frb}}$$



§. IUT (Main): Given initial Θ-data. [From LGP, (IUT$_3$)]

$\exists$ $\equiv$ a Θ-compatible s.t. "$\exists^{\text{indet}}$ r.s"

$\sim$ a/ Log-shell $\{\mathcal{I}_v\}_v$ $\curvearrowright$ $\prod \mathcal{I}_v$

b/ Θ-value $\{\underline{q}_v^{j}\}''$ $\curvearrowright$ $\prod \mathcal{I}_v$ $\Downarrow$

c/ #-field F_{mod} $\curvearrowright$ $\prod \mathcal{I}_v \otimes \mathbb{Q}$ $\Downarrow$

mod (Ind 1, 2)(Ind 3) - Ind

No interchange b/ & c/ (Lang)

In particular, wrt F_{mod}-portion of HT $\Theta^{+\log}$

$\mathbb{Q} > -\log|\Theta| > -\log|q|$

Region "$\mathcal{H}$" (Hoshi)

$\cup$ of q-pilot.

§ Beyond? (Coarse-hold)

IUT

•) Geom. content: $X \in \mathcal{M}_{11}(\mathbb{Z}/2\mathbb{Z})$ (Hol)

~ $X \in \mathcal{M}_{11}(\mathbb{Z}/6\mathbb{Z})$, $\mathbb{Z}/\ell$

$$\begin{array}{ccc} X & \dashrightarrow & X_v \\ \downarrow & & \\ C & \dashrightarrow & E_v \end{array}$$

② Simpl. $\pi_1^{tp}(X)$!

~ $\mathcal{M}_g(G) \longrightarrow \mathcal{M}_{11}(\mathbb{Z}/2)$

$X \longmapsto X/G$.

~ X_n = hyp. family of hyp. curves (Arak. OU)

•) Sym. & Galoisness: replace Kummer / $I_n \simeq \hat{\mathbb{Z}}(1)$

$I_{X,n} \longrightarrow \pi_1^{ab}(X)$ X 1-dim. orbifold?

•) Diophant. ∩ Anab. Geometry: Gr. Sect. Conj.

$$1 \to \pi_1(X_{\bar{k}}) \to \pi_1(X) \to G_k \to 1, \quad s: G_k \to \pi_1(X)$$

IUT: •) loc-glob. (Brauer) obst. for sect.: OU

•) Height ?

~ [Hoshi] Tomorrow

•) Abs. Anab. Geom. $\pi_1(X) \cdots \cancel{\rightarrow} \mathcal{G}_{k}$?!